Lecture 12

Single-component fluid

→ Relativistic matter

→ Non-relativistic matter

CMB temperature anisotropies

Single-component fluid

keep

$$-k^2\Phi - 3\frac{a'}{a}\Phi' - 3\frac{a'^2}{a^2}\Phi = 4\pi G a^2 \delta\rho \qquad (1)$$

$$\Phi'' + 3\frac{a'}{a}\Phi' + 2\left(\frac{a''}{a} - \frac{a'^2}{a^2}\right)\Phi = 4\pi G a^2 \delta p$$

$$(*) \quad \Phi'' + \underbrace{3\frac{a'}{a}(1+u_s^2)\Phi'}_{\text{friction}} + \underbrace{\left[2\frac{a''}{a} - \frac{a'^2}{a^2}(1-3u_s^2)\right]\Phi}_{\text{"mass term"}} + \underbrace{u_s^2k^2\Phi}_{\text{wave equation}} = 0$$

$u_s H^{-1}$ – sound horizon

(distance travelled by sound emitted after Big Bang using conservation we get

$$\left[2\frac{a''}{a} - \frac{a'^2}{a^2}(1-3u_s^2)\right] = -8\pi G a^2(p - u_s^2\rho)$$

$$= 0 \quad \text{if} \quad w = u_s^2$$

eq: (*) simplifies:

keep

$$\varphi'' + 3\frac{a'}{a}(1+u_s^2)\varphi' + u_s^2 k^2 \varphi = 0$$

take $\lambda \gg u_s H^{-1}$

$$\varphi'' + 3\frac{a'}{a}(1+u_s^2)\varphi'$$

$$\varphi'' + \frac{c}{\eta}\varphi' \rightarrow \varphi' = \text{const}$$

$$\rightarrow \varphi \sim \eta^{1-c} \xrightarrow[\eta\to\infty]{} 0$$

$$(c > 1)$$

Let's introduce relative density perturbation:

$$\delta = \frac{\delta\rho}{\rho}$$

$$(1): \quad \delta\rho \sim 3\frac{a'^2}{a^4}\varphi$$

$$\text{Freedman:} \quad \frac{a'^2}{a^4} = \frac{8\pi}{3}G\rho$$

$$\frac{\delta\rho}{\rho} = \text{const} \approx \approx \varphi_i$$

Relativistic Matter

In radiation domination $a \sim \eta$,

$$w = u^2 = 1/3$$

$$\varphi'' + \frac{4}{\eta}\varphi' + u_s^2 k^2 \varphi = 0$$

$\varphi(\eta)$, such that $\varphi(\eta) \xrightarrow[\eta\to 0]{} \text{const}$

(we match to the non-decaying mode)

$$\Phi(\eta) = -3\Phi_{(i)} \cdot \frac{1}{(u_s k\eta)^2}\left[\cos(u_s k\eta) - \frac{\sin(u_s k\eta)}{u_s k\eta}\right] \underset{k\to\infty}{\sim} \frac{\cos u_s k\eta}{(u_s k\eta)^2}$$

$$\underline{\underline{-k^2\varphi}} - 3\frac{a'}{a}\varphi' - 3\frac{a'^2}{a^2}\varphi = 4\pi G a^2 \delta\rho$$

dominates at $k \to \infty$

$$\delta\rho \simeq -\frac{1}{4\pi G}\frac{k^2}{a^2}\varphi$$

$$\delta_{rad} = \frac{\delta\rho_{rad}}{\rho_{rad}} = 6\,\Phi_i \cos u_s k\eta$$

No Jeans inst.

[perturbations of matter still grow logarithmically in RD]

Non-relativistic matter

$$\varphi'' + 3\frac{a'}{a}\left(1+\cancel{u_s^2}\right)\varphi' + \cancel{u_s^2 k^2\varphi} = 0$$

$\varphi = \text{const}$

$$\delta\rho = -\frac{1}{4\pi G a^2}\left(k^2 + \frac{12}{\eta^2}\right)\varphi$$

superhorizon: $\delta\rho \sim \dfrac{1}{a^3} \Rightarrow \dfrac{\delta\rho}{\rho} = \text{const}$

subhorizon: $\delta\rho \sim \dfrac{1}{a^2} \Rightarrow \underline{\underline{\dfrac{\delta\rho}{\rho} \sim a}}$

Jeans instability!

(it is power-law here)

This analysis is relevant for DM perturbations that enter horizon at matter domination [their present size is $\gtrsim$ few 100's Mpc. They have $\delta \sim 0.03$ $\rightarrow$ consistent with observations.

- Multi-component fluid

$\rightarrow$ Adiabatic and isocurvature modes:

Adiabatic: has $\delta\varphi$ deep in radiation domination (in our gauge) all δ_i's are the same

$\rightarrow$ Isocurvature: has $\delta\varphi = 0$

Most important is adiabatic

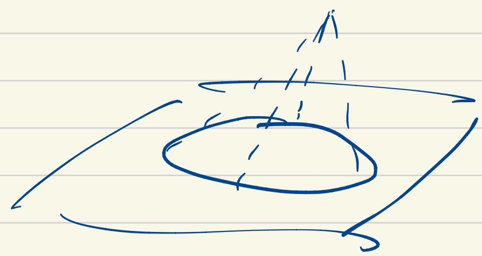
CMB anisotropies

(A sketch of the computation)

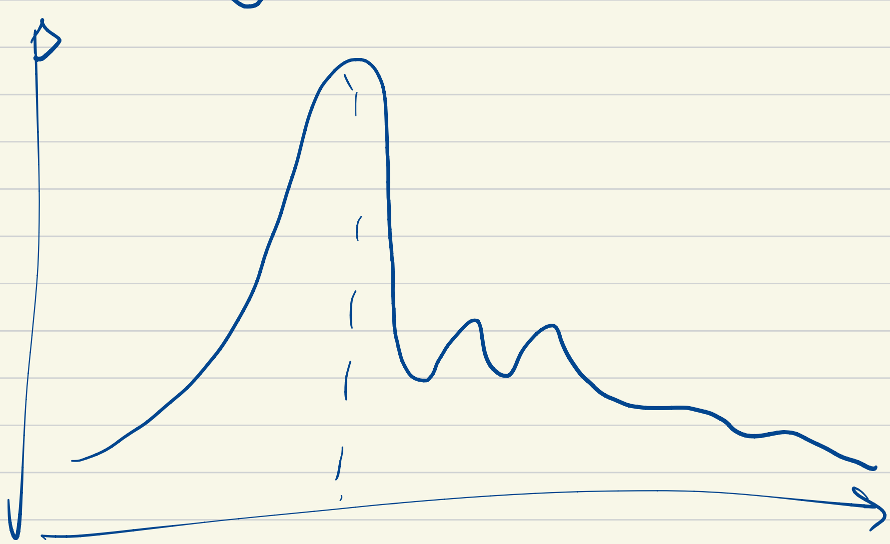
- Baryons and photons are tightly coupled before recombination. At recombination they decouple and we observe the photons.

• Photons coming from different directions have a slightly different temperature:

$$\frac{\delta T}{T}(\theta) \sim \delta_{B\gamma}(\theta) \qquad \theta \in S^2$$



In position space it looks random, however in "momentum" space there is a very clear pattern:



- This is the imprint of primordial perturbations.

$$\delta_{rad} = \frac{\delta \rho_{rad}}{\rho_{rad}} = 6\,\Phi_i(k) \cos u_s k \eta$$

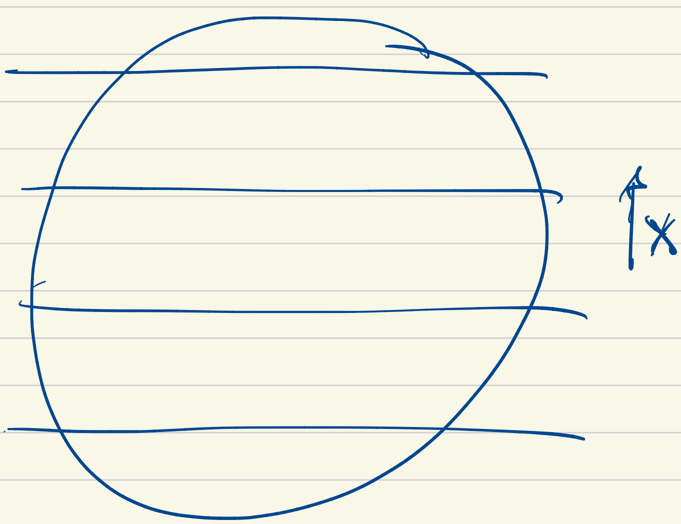
↳ primordial spectrum

$$\sim \frac{1}{k^{3/2}}$$

- oscillations

- Phases of different k are correlated.

- At $\eta \sim \eta_r$ some k's are excited and some are not.



Angular Fourier transform:

$$\int e^{ikx(\theta)} \sin 2\pi l \theta \, d\theta$$

$$\Delta\Theta_{max} \sim \frac{\eta_r}{\eta_0} \cdot \frac{1}{\sqrt{3}}$$

$$l_n \simeq n \frac{\pi}{\Delta\Theta} \simeq \sqrt{3}\,\pi \frac{\eta_0}{\eta_\sigma} \approx 300$$

$\frac{2\pi}{k}$ → coordinate wavelength

η_0 → coordinate distance traveled by a photon.